Diagnosis of Hybrid Systems with SMT: Opportunities and Challenges

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(Informal) Problem Definition

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Diagnosis

Detect, identify, and isolate faults in a system given observations of the system's behaviour.

Model-Based Diagnosis

A description of the system, i.e., a model, can be used to reason about the system's behaviour.

Hybrid Systems

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Dynamic Systems the state variables of whom can

- vary discretely (finite number of changes)

$$\forall [t, t'] \in \mathbf{R}^2.\ \exists k \in \mathbf{N}.\ \exists t = t_1 < \cdots < t_k = t'.$$

$$\forall \tau, \tau' \in]t_i, t_{i+1}[.\ v@\tau = v@\tau';$$

- vary continuously

$$\forall [t, t'] \in \mathbf{R}^2.\ \forall \nu \in]v@t, v@t'[\ \Rightarrow\ \exists t_\nu \in [t, t'].\ v@t_\nu = \nu;$$

- exhibit both types of behaviour.

Existing Approaches

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1. Separate the continuous aspects from the discrete ones: **loses interconnection between the variables**
   - Indicators on the continuous variables estimate the current discrete state
   - Discrete event systems techniques verify that the evolution of the discrete state is consistent with the model

2. Hybrid state tracking (particle filters, etc.): requires **predictive (probabilistic) models**

As opposed to diagnosis of DES, different approaches imply different models and different capabilities

Our Approach

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Diagnosis *à la* de Kleer, Reiter, Williams

Diagnostic Test

Verify the consistency between the model, the observations, and some assumption (reduced to BMC / SMT)

Diagnostic Algorithm

Generate the diagnostic tests in order to produce the diagnosis ($\rightarrow$ DX-11)

Satisfiability Modulo Theory

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SAT with an underlying theory

Examples of theories:

- bit-vectors and arrays,
- linear and non-linear arithmetics,
- recursive datatypes,
- default logic, etc.

We are interested in linear arithmetics:

$$(A \vee B) \wedge (x - y \geq 0) \wedge (\neg A \rightarrow (y < 9)) \wedge \ldots$$

Bounded Model Checking for Hybrid Systems

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Model-Checking

Verify reachability properties over hybrid systems (example: mutexes)

Bounded MC

Search for (counter-)examples that involve n (discrete and continuous) transitions

Reduction from Diagnosis Test to BMC

A diagnostic test is satisfiable iff there exists a path on the model that generates the observations and satisfies the assumption

Translating a Diagnostic Test into SMT

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Defining the SMT Variables

- For all state variable v and all timestep t, is defined a variable $\boxed{v@t}$

- For all timestep t, is defined a variable $\boxed{\texttt{time@}t}$

$\Rightarrow$ a timestep is an instant!

- For all event e and all odd timestep t, is defined a variable $\boxed{e@t}$

Translating a Diagnostic Test into SMT

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Discrete Variables

- For every timestep t,

$$\boxed{e@t \rightarrow \text{prec}(e)@t}$$

$$CB\_trip@t \rightarrow (current@t > 80)$$

- For every timestep t,

$$\boxed{e@t \rightarrow \text{effect}(e)@(t+1)}$$

$$CB\_trip@t \rightarrow open@(t+1)$$

- For every discrete state variable v,

$$\boxed{v@t \neq v@(t+1) \rightarrow \bigvee_{e \in \text{affecting}(v)} e@t}$$

$$(\neg open@t \wedge open@(t+1)) \rightarrow (CB\_trip@t \vee CB\_operated@t)$$

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Continuous Variables

- For every timestep t, for every continuous variable v,

$$\boxed{\texttt{time@}t = \texttt{time@}(t+1) \rightarrow v\texttt{@}t = v\texttt{@}(t+1)}$$

$$\texttt{time@}t = \texttt{time@}(t+1) \rightarrow \mathit{tpt}\texttt{@}t = \mathit{tpt}\texttt{@}(t+1)$$

- For every timestep t, for every continuous variable v,

$$\boxed{\mathrm{continuous\_constraint}(v, t, t+1)}$$

$$\mathit{tpt\_increasing}\texttt{@}t \rightarrow$$

$$((\mathit{tpt}\texttt{@}(t+1) - \mathit{tpt}\texttt{@}t) \geq 10 \times (\texttt{time@}(t+1) - \texttt{time@}t))$$

Translating a Diagnostic Test into SMT

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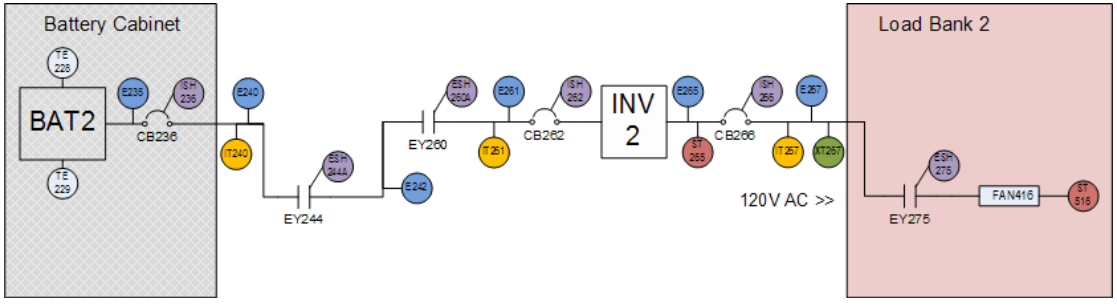
- State-based observations:
  $\boxed{obs\_variable@obs\_time = obs\_value}$
  (but the noise must be implemented into the model)

- Assumption: similar to diagnosis by SAT
  $\boxed{\neg f1\_occed@n \ \wedge \ f2\_occed@n \ \wedge \ \neg f3\_occed@n}$

Experiments

Adapt-Lite System



- 10 components
- 16 sensors
- 129 real-valued state variables
- 154 Boolean state variables

- 5-second windows (10 obs.)
- Preferred-First Strategy [DX11]
- SMT solver `Z3` version 4.3.1 (similar results with `cvc3`)

Results

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Prob. instance	Time (s)	Card	# δ
1	3.428	0	1
2	5.314	1	2
3	5.298	1	1
4	3.476	1	1
5	6.477	2	4

Results

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- Most of the runtime is on solving satisfiable problems
- Existing methods run faster but assume that fault patterns can be derived from the model
- Enormous scope for improvement:
  - Already significant improvement from DX-13
  - Simply removing redundant variables simplifies the SMT problems

Benefits of the Approach

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- Does not require a predictive model
- Is very flexible wrt. observations
- Justifies both diagnoses and non diagnoses

Second Round of Experiments

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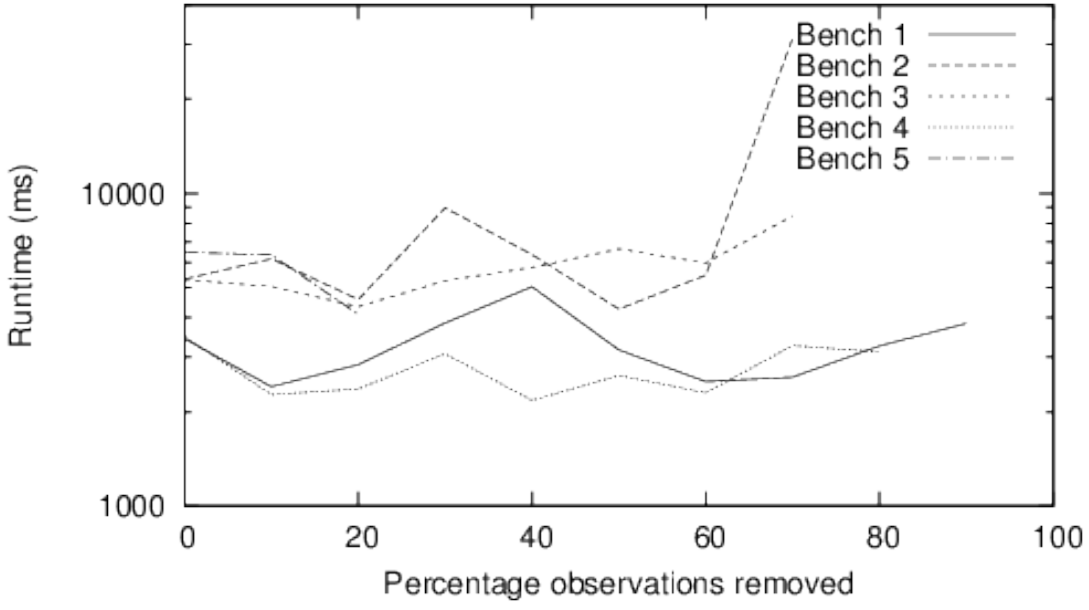
Existing methods rely on strong assumptions about observability
What happens when observability is variable?

Remove observations at random



Issues to Solve

- Improve performance: similar to Bounded-Model Checking or SAT planning
- Incremental computation (cf. work with Frank Su)

Conclusion

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- SMT techniques can be used to solve diagnosis problem of hybrid systems
  - First solution that integrates all the dimensions of the problem
  - Very flexible wrt model and observations
- Many problems remain to be addressed, but they are well-identified