Diagnosis (02)

Theory of Reiter

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Definition of Reiter Diagnosis

Naive algorithm

Second algorithm: diagnose

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Reiter Diagnosis

- A Reiter Diagnosis of an observed system $(SD, COMP, OBS)$ is a minimal set Δ such that $\Phi(\Delta)$ is a diagnosis
- **Remark**: a Reiter Diagnosis is a set of components and not a logical sentence but the representation are equivalent

Consequences of the definition

- First consequence

  $\emptyset$ is the only Reiter diagnosis of $(SD, COMP, OBS)$ iff $SD \cup OBS \cup \{\neg Ab(c) \mid c \in COMP\}$ is consistant

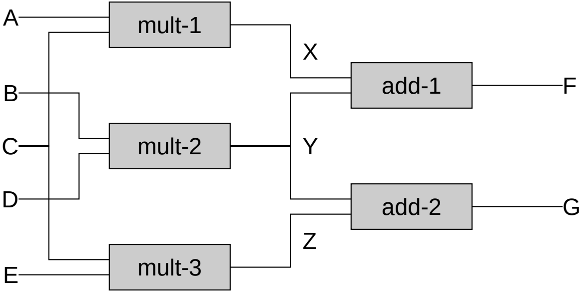
- Second consequence

  $\Delta \subseteq COMP$ is a Reiter diagnosis iff it is a minimal set such that $SD \cup OBS \cup \{\neg Ab(c) \mid c \in \Delta\}$ is consistant

- Problem: How to compute the Reiter diagnoses?

Example

What are the Reiter Diagnoses in this example?



Observations

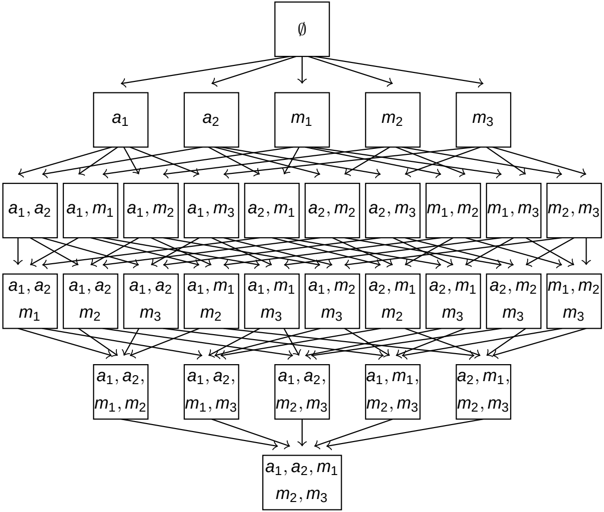
$In1(m_1, 3)$, $In2(m_1, 2)$, $In1(m_2, 2)$, $In2(m_2, 3)$
$In1(m_3, 2)$, $In2(m_3, 3)$, $Out(a_1, 10)$, $Out(a_2, 12)$

Definition of Reiter Diagnosis

Naive algorithm

Second algorithm: diagnose

Lattice of the states



Naive algorithm

- *Test and generate*
  - Explore the lattice from the root
  - Do not develop a node Δ of the lattice such that there exists $\Delta' \subset \Delta$ that is a Reiter diagnosis
  - Test if $SD \cup OBS \cup \{\neg Ab(c) \mid c \in \Delta\}$ is consistant
- Problem
  - Very inefficient

Definition of Reiter Diagnosis

Naive algorithm

Second algorithm: diagnose

Algorithm 2: Diagnose

- Rely on the notions of
  - Conflict
  - Hitting Set

Reiter conflict

- A Reiter conflict is a set of components $C \subseteq COMP$ that cannot all have a normal ehavior
- Formally: a set C is a Reiter conflict if

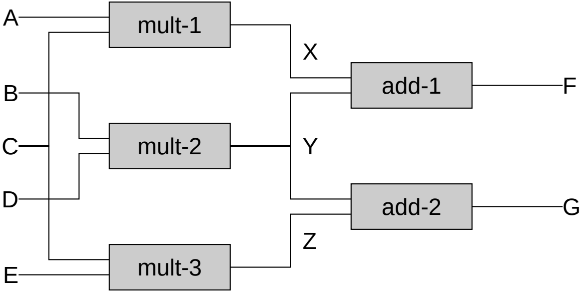
$$SD \cup OBS \cup \{\neg Ab(c) \mid c \in C\}$$

  is inconsistant

- A Reiter conflict C is minimal iff no subset of C is a Reiter conflict

Example

What Reiter Conflicts can you find in this example?



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From Reiter Conflicts to Reiter Diagnoses

Theorem

- $\Delta \subseteq COMP$ is a Reiter diagnosis for $(SD, COMP, OBS)$ iff $COMP \setminus \Delta$ is not a Reiter conflict

Hitting Set

Definition:

- Let $C = (S_1, \ldots, S_n)$ be a collection of sets
- H is a Hitting Set of C iff:
  - $H \subseteq \bigcup_{i \in \{1,\ldots,n\}} S_i$
  - $\forall i \in \{1, \ldots, n\}, H \cap S_i \neq \emptyset$

Example

- $C = \{\{2,4,5\},\{1,2,3\},\{1,3,5\},\{2,4,6\},\{2,4\}\}$
- Hitting sets:
  - $H_1 = \{2,1\}$
  - $H_2 = \{2,1,6\}$
  - $H_3 = \{4,3,2\}$
  - $H_4 = \{2,5\}$

Theorem

- $\Delta \subseteq COMP$ is a Reiter diagnosis for $(SD, COMP, OBS)$ iff Δ is a minimal Hitting Set for the set of the Reiter conflicts
- It is possible to consider the minimal Reiter conflicts

Naive algorithm using conflicts and hitting sets

- Compute the set F of all the minimal conflicts
- Compute the hitting set of F

- Problem: how to compute all the minimal conflicts?

Second solution

- Given a set $\Delta \subseteq COMP$, it is possible and *easy* to find one conflict $C \subseteq COMP \setminus \Delta$ using:
  - a demonstrator
  - the formula $SD \cup OBS \cup \{Ab(c) \mid c \in \Delta\}$

- Algorithm:
  1. Starting from $\Delta = \emptyset$
  2. if Δ is a Reiter Diagnosis (i.e. if the demonstrator did not find any conflict C), then stop
  3. otherwise, $\forall c \in C$, restart from 2 using $\Delta \cup \{c\}$

Example