Planning (06)

Planning as a SAT problem

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What is a SAT problem?

- A satisfiability problem is finding the values (in $\{1, 0\}$) of literals such that a proposition using these literals is true.
- Examples:
  - $(a \vee b) \wedge (\neg a \vee c) \wedge (\neg b \vee \neg a)$
  - A solution: $a = 1$, $b = 0$, $c = 1$

  - $(a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee \neg c) \wedge (\neg b \vee c) \wedge (a \vee \neg c)$
  - No solution!

  - $(a \vee b \vee c) \wedge (\neg a \vee \neg b) \wedge (\neg a \vee \neg c) \wedge (\neg b \vee \neg c)$
  - ???

Idea of planning as a SAT problem

Build a formula such that a solution is the plan

Variables of the SAT problem

Let n be the length of the plan we want to build

- For all $i \in \{0, \ldots, n\}$, for all literal l,
  l^i is the variable that indicates that l stands after the ith action
  - Example: $At(Jean\text{-}Luc, Le\ Mans)^0$, $EmptyClaw^2$, ...
- For all $i \in \{1, \ldots, n\}$, for all action a,
  a^i is the variable that indicates that the action a is performed that the date i
  - Example: $Go(Didier, Rennes)^1$, $Take(A, B)^3$, ...

How many actions?

- Problem 1: we don't know the value of n
- Problem 2: the complexity of the computation (and of the solution!) increases with n

- Solution:
  - We start with $n = 0$ and try to find a solution
  - We increase n until we find a solution
  - If there is no solution, then we have to increase n to $2^{nb\_literals}$ which is too big

How to build the proposition?

- General formula:

$$\mathit{Initial\_state\_formula} \wedge \mathit{Transitions\_formula} \wedge \mathit{Goal\_formula}$$

- Formula that represents the initial state:

$$\bigwedge_{\forall l \in \mathit{Initial\_state}} l^0 \wedge \bigwedge_{\forall l \notin \mathit{Initial\_state}} \neg l^0$$

- Formula that represents the goal states:

$$\bigwedge_{\forall l \in \mathit{Goal}} l^n$$

Transition formula (1)

The transition formula is a conjunction of formulae

- The precondition $p(a)$ of an action must be true before applying the action a

$$\forall a \in \mathcal{A}, \forall i \in \{1, \ldots, n\}, \qquad a^i \Rightarrow \bigwedge_{l \in p(a)} l^{i-1}$$

- Example:
  - action $Take(A, B)$
  - precondition $OnTop(A) \wedge On(A, B) \wedge EmptyClaw$
  - $Take(A, B)^1 \Rightarrow OnTop(A)^0 \wedge On(A, B)^0 \wedge EmptyClaw^0$

Transition formula (2)

- The effect $e(a)$ of an action must be true after applying the action a

$$\forall a \in \mathcal{A}, \forall i \in \{1, \dots, n\}, \qquad a^i \Rightarrow \bigwedge_{l \in e(a)} l^i$$

- Example:
  - action $Take(A, B)$
  - effect $Holding(A) \wedge OnTop(B) \wedge \neg OnTop(A) \wedge \neg On(A, B) \wedge \neg EmptyClaw$
  - $Take(A, B)^1 \Rightarrow Holding(A)^1 \wedge OnTop(B)^1 \wedge \neg OnTop(A)^1 \wedge \neg On(A, B)^1 \wedge \neg EmptyClaw^1$

Transition formula (3)

- Frame axiom: the value of a literal is modified only by a transition

$$\forall l, \forall i \in \{1, \ldots, n\}, \qquad l^{i-1} \wedge \neg l^{i} \Rightarrow \Big(\bigvee_{a \mid \neg l \in Effect(a)} a^{i} \Big)$$

- Example:
  - literal l
  - the actions that set l to false are a and b
  - $l^0 \wedge \neg l^1 \Rightarrow (a^1 \vee b^1)$

- Similarly

$$\forall l, \forall i \in \{1, \ldots, n\}, \qquad \neg l^{i-1} \wedge l^{i} \Rightarrow \Big(\bigvee_{a \mid l \in Effect(a)} a^{i} \Big)$$

Transition formula (4)

- One and exactly one action is performed at one date

$$\forall i \in \{1, \ldots, n\}, \qquad \bigvee_{a \in \mathcal{A}} a^i$$

$$\forall i \in \{1, \ldots, n\}, \forall \{a, b\} \subseteq \mathcal{A}, a \neq b, \qquad \neg a^i \vee b^i$$

- Example
  - There are three actions a, b and c
  - $a^1 \vee b^1 \vee c^1$
  - $\neg a^1 \vee \neg b^1$
  - $\neg a^1 \vee \neg c^1$
  - $\neg b^1 \vee \neg c^1$

Example – Problem model

- Two literals describing the state: l, k
- Initial state $\{l\}$
- Goal: $\{k\}$
- Actions:
  - a: $p(a) = l$, $e(a) = \{k\}$
  - b: $p(b) = \{k, l\}$, $e(b) = \{\neg k, \neg l\}$

Example – SAT problem

We consider $n = 1$